May 2003

Noise Trader Risk and the Welfare Effects of Privatization*

Abstract

Excessive volatility of asset prices like that generated in the 'noise trader' model of De Long et al. is one factor that plausibly might contribute to an explanation of the equity premium. We extend the De Long et al. model to allow for privatization of publicly-owned assets and assess the welfare effects of such privatization in the presence of excess volatility arising from noise traders' mistaken beliefs.

JEL Classification: E62

Key words: equity premium, noise trader risk, privatization.

Simon Grant
Department of Economics
Rice University

John Quiggin
School of Economics
Queensland University

*Financial support for this project has been provided by the Australian Research Council's Grant A79800678. Quiggin also gratefully acknowledges income support from an ARC Professorial Fellowship. This paper has also benefitted from many useful comments and suggestions of two referees.

1 Introduction

Privatization of public assets has been a widely-adopted policy in recent years. In addition to the widespread privatization accompanying the downfall of Communism, there have been numerous privatizations in both developed and less developed countries. Privatization has been advocated on a number of grounds, including improvements in operating efficiency and the desire to subject managers to the discipline of takeover markets.

An important element of the case for privatization is the claim that the sale of publicly-owned assets permits an improvement in the financial position of governments, and, in particular, a reduction in public debt. This argument has been prominent in the advocacy of privatization by the World Bank, summarized by Kikeri, Nellis and Shirley (1992). This motivation is particularly important in the case of partial privatization, where governments retain majority ownership, so that there is no reason to anticipate changes in operating efficiency or beneficial effects of capital market discipline.

Despite the frequency with which privatization is recommended as a fiscal expedient, it is frequently the case, particularly in developed countries, that the sale proceeds realized through privatization are less than the expected earnings of the enterprise under continued public ownership, discounted at the real bond rate (Vickers and Yarrow 1988, Quiggin 1995). In some cases, particularly where privatization is undertaken through an initial public offering of stock, the difference is partly due to the fact that the offer price is well below the market price revealed on the first day of trading. The case of British Telecom, examined by Vickers and Yarrow, is illustrative.

A more fundamental reason for the divergence between sale prices and future earnings is the substantial difference, referred to as the equity premium, between the rate of return demanded by holders of equity and the rate of return demanded by the holders of government bonds. As was first observed by Mehra and Prescott (1985), the magnitude of the equity premium is a puzzle, since application of the standard consumption capital asset pricing model (CCAPM) with plausible parameters suggests that the premium should be less than one percentage point. By contrast, typical empirical estimates of the equity premium are around six percentage points.

Kocherlakota (1996) surveys a large number of papers in which attempts are made to explain the large observed values of the equity premium and concludes that 'it's still a puzzle'. Although the explanations surveyed by Kocherlakota differ in many respects, all of them are based on the assumption that holdings of equity can be regarded simply as claims to a particular proportion of corporate profits, with no account being taken of the properties of the stock markets in which equities are bought and sold. In the terminology of Hirshleifer and Riley (1992) these models only deal with event uncertainty and do not consider issues of market uncertainty such as the optimal search for trading partners or disequilibrium processes and price dynamics. Implicitly, some version of the efficient markets hypothesis is assumed to apply to stock markets, although failures in other financial markets (such as insurance markets) are postulated in some cases.

The work of Shiller (1989) suggests an alternative approach to the equity premium puzzle. If, as Shiller argues, financial markets display excess volatility, then returns to holdings of equity are riskier than are the associated streams of corporate profits. Shiller's insight has been formalized in the 'noise trader' model of De Long et al. (1990). In this model, risk over and above that due to the dividend-generating process is introduced into the economy by the distorted and stochastic, beliefs of misinformed investors referred to as 'noise traders'. De Long et al. observe that this excess risk implies an increase in the equity premium relative to the case when all investors have rational expectations, but they do not consider the policy implications of this observation. De Long et al. show that, although both noise traders and sophisticated investors are made better off in *ex ante* terms (given their beliefs) by the availability of trade, this apparent welfare improvement arises at the expense of those holding equity when trade is introduced, such as entrepreneurs making initial public offerings.

A large number of subsequent writers have developed the work of Shiller (1989) and De Long et al. (1990) on market volatility. Although the use of terms like 'excess volatility' implies some departure from efficiency and therefore some potential policy implications, these issues have received relatively little attention.[1] In particular, the implications of volatility generated by noise traders for the appropriate risk premium for public investments, and for the welfare and distributional effects of privatization, have not been considered.

In this paper, we address the latter issue. We modify the De Long et al. model to allow for the existence of an asset that is initially publicly owned, but is otherwise similar to the private asset considered by De Long et al. We then examine the consequences of privatization for asset prices and demands, and for the welfare of different groups. We show that if the equity premium arises from the mistaken beliefs of noise traders, privatization may reduce public sector net worth. Moreover, if the noise-traders' misperceptions about the returns of existing equity and the privatized asset are positively correlated then, evaluated in terms of the correct beliefs of sophisticated investors, there is a reduction in social welfare associated with privatization.

2 The Analysis

Following De Long et al. (1990) we introduce a stripped-down overlapping generations model with two-period lived agents. There is a single consumption good but there is no consumption when young, no labor supply decision and no bequest motive. The only decision an agent makes is her choice of portfolio when young to finance her consumption when old.

[1] Orosel (1996) discusses potential adverse welfare implications arising from the availability of public information in stock markets. Palley (1999) considers the implications of a 'noise trader' model for taxes on international financial transactions.

There is no fundamental risk and all assets pay a fixed real dividend r. One asset, the *safe* asset, is in perfectly elastic supply. Any unit of the safe asset can be converted into one unit of the consumption good, and vice versa. As De Long et al. note, the safe asset is formally equivalent to a storage technology that pays a real net return of r. Furthermore, if we take the consumption good in each period as the numeraire, the price of the safe asset is always one. A second asset, that we shall interpret as the *pre-privatization* economy-wide portfolio of equity, is in fixed supply, normalized to one. The price of this equity asset in period t is denoted by p_t^e. De Long et al. point out that if the price of the equity asset were simply the net present value of its future dividends, then its price in every period would also be 1. But, in the presence of noise traders, De Long et al. show that this is not the case.

Extending De Long et al., we introduce a third asset that is also in fixed supply, x, and that generates a real dividend r. Initially, this asset is owned by the government and financed entirely through short-term (one-period) government debt. Government debt, the fourth asset in our model, pays a guaranteed fixed real interest r. As government debt is a perfect substitute for the safe asset, its price in every period is always one.

Every generation is the same size and can be divided into two classes: a proportion λ who are noise traders (denoted N) whose behavior is described in more detail in subsequent subsections below, and a proportion $1-\lambda$ who are sophisticated investors (denoted I). In each period, the representative sophisticated investor has rational expectations about the distribution of returns from holding a portfolio with risky assets, and so maximizes her expected utility given the distribution of her wealth implied by her portfolio choice.

2.1 The Pre-Privatization Equilibrium

For any period t in which the third asset remains in government ownership, the government issues x units of new debt (of one-period maturation) which is purchased by individuals who are young in period t. Using the proceeds of this bond sale together with the real dividend generated by the government-owned asset, the government pays out the amount $(1+r)\,x$ to the holders of the x units of government debt that was issued in period $t-1$ and that has matured in period t.

In each period t, the representative noise trader who is young in that period misperceives the expected price of the asset in period $t+1$ by an independent and identically normally-distributed random variable

$$d_t^e \sim N\left(d^*, \sigma_d^2\right).$$

We assume that both sophisticated investors and noise traders are expected utility maximizers characterized by a constant coefficient of absolute risk aversion equal to γ. Thus, an agent who is young in period t, chooses her portfolio to maximize her *certainty equivalent* *consumption* in period $t+1$. That is, she maximizes

$$CE = \overline{w} - \gamma\sigma_w^2/2,$$

where $\overline{w}$ is the expected final wealth in period $t+1$, and σ_w^2 is the variance of her period $t+1$ wealth.

De Long et al. (1990) show that, in a stationary equilibrium, that is, where one imposes the requirement that the unconditional distribution of p_{t+1}^e be identical to the distribution of p_t^e, the pricing rule for equity takes the form

$$p_t^e = 1 + \frac{\lambda\left(d_t^e - d^*\right)}{1+r} + \frac{\lambda d^*}{r} - \frac{\gamma\lambda^2\sigma_d^2}{r\left(1+r\right)^2}, \tag{1}$$

where $\lambda^2\sigma_d^2/\left(1+r\right)^2$ is the constant one-period-ahead variance of p_t^e.[2]

De Long et al. (1990, p. 731) observe "[i]t is important to stress that our model sheds light on the Mehra–Prescott puzzle only if equities are underpriced, which is itself a necessary condition for noise traders to earn higher expected returns." They (and we) view the existence of the equity premium as suggestive that noise traders are on average bullish and may earn higher average returns than sophisticated traders, that is, arbitrageurs. Hence we shall assume both that noise traders are bullish, $d^* > 0$, and that equities are underpriced, that is, $\mathrm{E}\left[p_t^e\right] < 1$, or equivalently,

$$\frac{\gamma\lambda\sigma_d^2}{\left(1+r\right)^2} - d^* > 0. \tag{2}$$

2.2 The Post-Privatization Equilibrium

We shall now consider the situation where the government announces at the beginning of period 0 that it is privatizing the third asset which it has held in government ownership up to that date. The amount $\left(1+r\right)x$ owing on the outstanding stock of government bonds which are held by the current old generation, will be paid out of the dividend xr generated by the asset and the revenue $p_0^{ne}x$ generated by the sell-off of the x units of supply of this asset at the price p_0^{ne}. Any shortfall (respectively, windfall) will be met by a stream of higher (respectively, lower) taxes on the young in each subsequent period that has the same net present value as the shortfall (respectively, windfall).

As a natural generalization of the De Long et al. model, we assume that the misperceptions for each period t (for $t \geq 0$) of the expected price of equity and the expected price of the privatized asset (the "new equity") in period $t+1$ are independently and identically distributed as bivariate normal

$$\begin{pmatrix} d_t^e \\ d_t^{ne} \end{pmatrix} \sim N\left(d^*\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \sigma_d^2\begin{pmatrix} 1 & \beta \\ \beta & 1 \end{pmatrix}\right).$$

[2] See De Long et al. (1990) pp. 708–11 for the derivation.

We assume that the government has to make its decision to privatize the asset *before* d_0^e and d_0^{ne} are realized.

The term β represents the correlation between misperceptions of the prices of existing private equity and of the newly privatized asset. To the extent that misperceptions relate to idiosyncratic risk associated with particular enterprises, β will be low. Conversely, to the extent that misperceptions relate to systematic risk, β will be near 1. The explanation of the equity premium provided by De Long et al. requires substantial misperception of systematic risk for private equity, suggesting that β should be near 1.

To aid the exposition let us introduce the following notations:

$$\begin{aligned}\mathbf{d}_t &= \begin{pmatrix} d_t^e \\ \\ d_t^{ne} \end{pmatrix}, \mathbf{p}_t = \begin{pmatrix} p_t^e \\ \\ p_t^{ne} \end{pmatrix}, \boldsymbol{\mu}_t = \begin{pmatrix} \mu_t^e \\ \\ \mu_t^{ne} \end{pmatrix} = \mathrm{E}_t \left[\begin{pmatrix} p_{t+1}^e \\ \\ p_{t+1}^{ne} \end{pmatrix} \right], \\ \boldsymbol{\Sigma}_t &= \left[\mathrm{E}_t \left[\left(p_{t+1}^i - \mu_t^i \right) \left(p_{t+1}^j - \mu_t^j \right) \right]_{i,j=e,ne} \right] \text{ and } \mathbf{1} = \begin{pmatrix} 1 \\ \\ 1 \end{pmatrix}.\end{aligned}$$

Consider a sophisticated investor with a pre-tax amount of labor income y_t facing a tax liability of τ_t. Her objective is to choose a portfolio $\mathbf{q}_I^{\mathrm{T}} = (q_I^e, q_I^{ne})$, where q_I^e is her holding of equity and q_I^{ne} is her holding of the privatized asset, and the remainder of her post-tax income $(y_t - \tau_t - \mathbf{p}_t^{\mathrm{T}} \mathbf{q}_I)$ is invested in the safe asset and/or government bonds. Her optimal portfolio choice maximizes:

$$(y_t - \tau_t)(1+r) + \left[r\mathbf{1} + \boldsymbol{\mu}_t - (1+r)\mathbf{p}_t \right]^{\mathrm{T}} \mathbf{q}_I - \frac{\gamma}{2} \mathbf{q}_I^{\mathrm{T}} \boldsymbol{\Sigma}_t \mathbf{q}_I. \tag{3}$$

Similarly, the representative noise trader with an amount $y_t - \tau_t$ to invest in period t, chooses a portfolio $\mathbf{q}_N^{\mathrm{T}} = (q_N^e, q_N^{ne})$ that maximizes

$$(y_t - \tau_t)(1+r) + \left[r\mathbf{1} + \boldsymbol{\mu}_t - (1+r)\mathbf{p}_t \right]^{\mathrm{T}} \mathbf{q}_N - \frac{\gamma}{2} \mathbf{q}_N^{\mathrm{T}} \boldsymbol{\Sigma}_t \mathbf{q}_N + \mathbf{d}_t^{\mathrm{T}} \mathbf{q}_N. \tag{4}$$

The only difference between (3) and (4) is the last term of (4) which reflects the noise traders' misperceptions of the expected returns from holding equity and from holding the privatized asset.

The corresponding first order conditions derived from (3) and (4) yield the asset demands

$$\mathbf{q}_I = \frac{1}{\gamma} \boldsymbol{\Sigma}_t^{-1} \left[r\mathbf{1} + \boldsymbol{\mu}_t - (1+r)\mathbf{p}_t \right] \tag{5}$$

$$\mathbf{q}_N = \frac{1}{\gamma} \boldsymbol{\Sigma}_t^{-1} \left[r\mathbf{1} + \boldsymbol{\mu}_t - (1+r)\mathbf{p}_t + \mathbf{d}_t \right]. \tag{6}$$

Solving for the market-clearing prices we obtain

$$\mathbf{p}_t = \frac{1}{1+r} \left[r\mathbf{1} + \boldsymbol{\mu}_t + \lambda \mathbf{d}_t \right] - \frac{\gamma}{1+r} \Sigma_t \begin{pmatrix} 1 \\ \\ x \end{pmatrix}. \tag{7}$$

We confine attention to steady-state equilibria by imposing the requirement that the unconditional distribution of $\mathbf{p}_{t+1}$ be identical to the distribution of $\mathbf{p}_t$. Analogous to the method used by De Long et al. (see pp710-11), we can solve (7) recursively to obtain

$$\mathbf{p}_t = \mathbf{1} + \frac{\lambda}{1+r}\left(\mathbf{d}_t - d^*\mathbf{1}\right) + \frac{\lambda d^*}{r}\mathbf{1} - \frac{\gamma}{r}\Sigma_t \begin{pmatrix} 1 \\ x \end{pmatrix}. \tag{8}$$

Inspection of (8) reveals a time-invariant variance–covariance matrix for $\mathbf{p}_t$ of

$$\Sigma = \frac{\lambda^2 \sigma_d^2}{(1+r)^2} \begin{pmatrix} 1 & \beta \\ \beta & 1 \end{pmatrix}$$

That is, we have

$$p_t^e = 1 + \frac{\lambda\left(d_t^e - d^*\right)}{1+r} + \frac{\lambda d^*}{r} - \frac{\gamma \lambda^2 \sigma_d^2}{r(1+r)^2}(1+\beta x), \tag{9}$$

$$p_t^{ne} = 1 + \frac{\lambda\left(d_t^{ne} - d^*\right)}{1+r} + \frac{\lambda d^*}{r} - \frac{\gamma \lambda^2 \sigma_d^2}{r(1+r)^2}(\beta + x). \tag{10}$$

Notice that for small x, the correlation coefficient β is approximately equal to the 'beta' coefficient that would come out of the standard CAPM. More significantly, since preferences display constant absolute risk aversion, and since the sequence of taxes τ_t is determined in period 0, the level of labor income in any period and the distribution of labor-income taxes between taxpayers in future periods have no impact on asset demands or equilibrium prices.

2.3 Welfare Effects of the Privatization

The intertemporal production technology for the consumption good implicitly embodies a real net return of r. Hence, welfare changes across generations in this economy can be characterized in terms of the net present value of the changes in consumption streams using a discount rate of r.

For $t \geq 0$, let $\Delta\,\mathrm{CE}_t^I$ (respectively, $\Delta\,\mathrm{CE}_t^N$) denote for the representative sophisticated investor (respectively, noise-trader) who is young in period $t-1$, the change that results from the privatization in his or her certainty equivalent consumption in period t. If we let ΔW denote the net present value of the changes in the certainty equivalent consumption of every generation, then the *ex ante* change is given by

$$\mathrm{E}\left[\Delta W\right] = \sum_{t=0}^{\infty} \frac{(1-\lambda)\,\mathrm{E}\left[\Delta\,\mathrm{CE}_t^I\right] + \lambda\,\mathrm{E}\left[\Delta\,\mathrm{CE}_t^N\right]}{(1+r)^t} \tag{11}$$

Let $\overline{z}$ denote the value the variable z would have taken if the government had not privatized the asset in period 0. We consider the changes to present and future consumers.

Consumers in period 0.

These consumers have already made their portfolio choice in the previous period. The only action they undertake in period 0 is to sell their portfolio on the market to finance their consumption. Inspection of (1) and (9) reveals that $\overline{p}_0^e$ and p_0^e have the same variance. Hence it follows that:

$$(1-\lambda)\,\mathrm{E}\left[\Delta\,\mathrm{CE}_0^I\right] + \lambda\,\mathrm{E}\left[\Delta\,\mathrm{CE}_0^N\right] = \mathrm{E}\left[p_0^e - \overline{p}_0^e\right] = -\frac{\gamma\lambda^2\sigma_d^2}{r\,(1+r)^2}\beta x,$$

which means that consumers in period 0 will lose (respectively, gain) if the noise traders' misperception of the expected price of the newly privatized asset in the next period is positively (respectively, negatively) correlated with their misperception of the expected price of the old equity in the next period. As a referee has pointed out, for any financial innovation, holders of existing assets will gain if the new asset's return is negatively correlated with the existing stock market as the price of the existing stocks increases since investors can better diversify their portfolio. Here the only source of uncertainty is the misperception of the noise traders.

Sophisticated consumers in period $t \geq 1$.

For the sophisticated investor who will consume in period t we have

$$\begin{aligned}\Delta\,\mathrm{CE}_t^I &= \left[r\mathbf{1} + \boldsymbol{\mu}_{t-1} - (1+r)\,\mathbf{p}_{t-1}\right]^{\mathrm{T}}\mathbf{q}_I - \frac{\gamma}{2}\mathbf{q}_I^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{q}_I \\ &- \left(r + \mathrm{E}_{t-1}\left[\overline{p}_t^e\right] - (1+r)\,\overline{p}_{t-1}^e\right)\overline{q}_I^e + \frac{\gamma}{2}\frac{\lambda^2\sigma_d^2}{(1+r)^2}\left(\overline{q}_I^e\right)^2 - (1+r)\,E\left[\Delta\tau_{t-1}\right]\end{aligned}$$

where $\Delta\tau_t$ denotes the change in taxation of labor income of the young in period $t \geq 1$ that arises as a result of the privatization and where $\Delta\tau_0 = 0$.

In the appendix, we show this may be expressed as

$$\Delta\,\mathrm{CE}_t^I = \frac{(1+r)^2}{2\gamma\sigma_d^2} \times \frac{\left(\beta d_{t-1}^e - d_{t-1}^{ne}\right)^2}{\left(1-\beta^2\right)} + \lambda x\left(\frac{\gamma\lambda\sigma_d^2}{(1+r)^2}\left(\beta + \frac{x}{2}\right) - d_{t-1}^{ne}\right) - (1+r)\,E\left[\Delta\tau_{t-1}\right].$$

Thus,

$$\mathrm{E}\left[\Delta\,\mathrm{CE}_t^I\right] = \frac{(1+r)^2\left((1-\beta)\left(d^*\right)^2 + (1+\beta)\,\sigma_d^2\right)}{2\gamma\,(1+\beta)\,\sigma_d^2} + \lambda x\left(\frac{\gamma\lambda\sigma_d^2}{(1+r)^2}\left(\beta + \frac{x}{2}\right) - d^*\right) - (1+r)\,E\left[\Delta\tau_{t-1}\right]. \tag{12}$$

For a noise trader who will consume in period t we have two possible measures of the impact on welfare. Expected utility may be calculated either with respect to the true distribution or with respect to the distorted distribution that forms the basis of asset trading decisions. Depending on the perspective of the decision-maker seeking to evaluate welfare, either or both may be relevant.

Noise consumers' expected utility with respect to perceived distribution

The objective function that guides asset purchases decisions for noise consumers is based on the perceived distribution. The impact of privatization on this objective function is given by:

$$
\begin{aligned}
&\Delta\,\mathrm{CE}_t^N = \left[r\mathbf{1} + \boldsymbol{\mu}_{t-1} - (1+r)\,\mathbf{p}_{t-1} + \mathbf{d}_{t-1}\right]^{\mathrm{T}} \mathbf{q}_N - \frac{\gamma}{2}\mathbf{q}_N^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{q}_N \\
&- \left(r + \mathrm{E}_{t-1}\left[\overline{p}_t^e\right] - (1+r)\,\overline{p}_{t-1}^e + d_{t-1}^e\right)\overline{q}_N^e + \frac{\gamma}{2}\frac{\lambda^2\sigma_d^2}{(1+r)^2}\left(\overline{q}_N^e\right)^2 - (1+r)\,E\left[\Delta\tau_{t-1}\right] \\
= \;&\frac{(1-\lambda)^2}{\lambda^2} \times \frac{(1+r)^2}{2\gamma\sigma_d^2} \times \frac{\left(\beta d_{t-1}^e - d_{t-1}^{ne}\right)^2}{\left(1-\beta^2\right)} + x\left(\frac{\gamma\lambda^2\sigma_d^2}{(1+r)^2}\left(\beta + \frac{x}{2}\right) + (1-\lambda)\,d_{t-1}^{ne}\right) - (1+r)\,E\left[\Delta\tau_{t-1}\right].
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\mathrm{E}\left[\Delta\,\mathrm{CE}_t^N\right] \;=\; &\frac{(1-\lambda)^2}{\lambda^2}\frac{(1+r)^2\left((1-\beta)\,(d^*)^2 + (1+\beta)\,\sigma_d^2\right)}{2\gamma\,(1+\beta)\,\sigma_d^2} \\
&+x\left(\frac{\gamma\lambda^2\sigma_d^2}{(1+r)^2}\left(\beta + \frac{x}{2}\right) + (1-\lambda)\,d^*\right) - (1+r)\,E\left[\Delta\tau_{t-1}\right]. \qquad (13)
\end{aligned}
$$

Hence the per capita change in the *ex ante* certainty equivalent consumption for the generation who will be consumers in period t (≥ 1) is

$$
(1-\lambda)\,\mathrm{E}\left[\Delta\,\mathrm{CE}_t^I\right] + \lambda\,\mathrm{E}\left[\Delta\,\mathrm{CE}_t^N\right]
$$

$$
= \frac{(1-\lambda)}{\lambda}\frac{(1+r)^2\left((1-\beta)\,(d^*)^2 + (1+\beta)\,\sigma_d^2\right)}{2\gamma\,(1+\beta)\,\sigma_d^2} + x\left(\frac{\gamma\lambda^2\sigma_d^2}{(1+r)^2}\left(\beta + \frac{x}{2}\right)\right) - (1+r)\,E\left[\Delta\tau_{t-1}\right] \qquad (14)
$$

The first term measures the perceived benefits from the diversification opportunities afforded by the introduction of trade in the privatized asset, the second term reflects the perceived sum of ex ante gains the noise and sophisticated traders anticipate from their net positions. These gains are offset (respectively, augmented) by the share of any shortfall (respectively, windfall) of the government's financial position arising from the privatization.

This yields in net present value terms

$$
\mathrm{E}\left[\Delta W\right] = \frac{(1-\lambda)\,(1+r)^2\left((1-\beta)\,(d^*)^2 + (1+\beta)\,\sigma_d^2\right)}{2r\lambda\gamma\,(1+\beta)\,\sigma_d^2} + \frac{\gamma\lambda^2\sigma_d^2x^2}{2r\,(1+r)^2} - \sum_{t=1}^{\infty}\frac{E\left[\Delta\tau_t\right]}{(1+r)^t} \qquad (15)
$$

Noise consumers' expected utility with respect to true distribution

An alternative measure of the welfare effects of the privatization is to evaluate the expected utility of every agent with respect to the *true* distribution of consumption. The only agents this affects are the noise traders who will be consuming in periods $1, 2, \ldots$. Let $\Delta\widehat{\mathrm{CE}_t^N}$ denote this value.

$$
\Delta\widehat{\mathrm{CE}_t^N} = \Delta\,\mathrm{CE}_t^N - \left(\mathbf{d}_{t-1}^{\mathrm{T}}\mathbf{q}_N - d_{t-1}^e\overline{q}_N^e\right) - (1+r)\,E\left[\Delta\tau_{t-1}\right]
$$

Recall

$$\mathbf{q}_N = \frac{1}{\gamma}\Sigma^{-1}\left[r\mathbf{1} + \boldsymbol{\mu}_{t-1} - (1+r)\,\mathbf{p}_{t-1} + \mathbf{d}_{t-1}\right].$$

From (7) we have

$$r\mathbf{1} + \boldsymbol{\mu}_{t-1} - (1+r)\,\mathbf{p}_{t-1} + \mathbf{d}_{t-1} = \gamma\Sigma\begin{bmatrix} 1 \\ \\ x \end{bmatrix} + (1-\lambda)\,\mathbf{d}_{t-1},$$

and hence

$$\mathbf{d}_{t-1}^{\mathrm{T}}\mathbf{q}_N - d_{t-1}^{e}\overline{q}_N^{e} = xd_{t-1}^{ne} + \frac{(1-\lambda)(1+r)^2}{\gamma\lambda^2\sigma_d^2\left(1-\beta^2\right)}\left(\beta^2\left(d_{t-1}^{e}\right)^2 - 2\beta d_{t-1}^{e}d_{t-1}^{ne} + \left(d_{t-1}^{ne}\right)^2\right).$$

This gives us a corrected measure of the change in *ex ante* welfare of

$$\mathrm{E}\left[\Delta\widehat{W}\right] = -\frac{\lambda x}{r}\left(d^* - \frac{\gamma\lambda\sigma_d^2 x}{2(1+r)^2}\right) - \frac{(1-\lambda)(1+r)^2\left((1-\beta)(d^*)^2 + (1+\beta)\sigma_d^2\right)}{2r\lambda\gamma(1+\beta)\sigma_d^2} - \sum_{t=1}^{\infty}\frac{E\left[\Delta\tau_t\right]}{(1+r)^t} \tag{16}$$

Notice that the difference between (15) and (16) is

$$\frac{\lambda x}{r}d^* + \frac{(1-\lambda)(1+r)^2\left((1-\beta)(d^*)^2 + (1+\beta)\sigma_d^2\right)}{r\lambda\gamma(1+\beta)\sigma_d^2}.$$

The first term reflects the extra consumption that noise traders' misperceptions lead them to expect, on average, to receive by holding the privatized asset while the second term reflects the certainty-equivalent consumption cost of the risk associated with taking bets based on misperceived expected future prices of the privatized asset.

Finally, we may evaluate the expected net present value of the tax change as

$$\begin{aligned}
\sum_{t=1}^{\infty}\frac{E\left[\Delta\tau_t\right]}{(1+r)^t} &= \left(x(1+r) - xr - \mathrm{E}\left[p_0^{ne}\right]x\right) \\
&= x\left(1 - \mathrm{E}\left[p_0^{ne}\right]\right) \\
&= \frac{\lambda x}{r}\left(d^* - \frac{\gamma\lambda\sigma_d^2}{(1+r)^2}(\beta + x)\right).
\end{aligned} \tag{17}$$

Assuming that condition (2) is satisfied, so that there is a positive equity premium, the government will be worse off whenever $\beta + x$ is sufficiently close to or greater than 1. Notice that although this is more likely to hold the larger is the scale of the privatization relative to the existing equity market (i.e. the larger is x), even a small scale privatization may worsen the government's financial position, if the representative noise-trader's misperceptions of the expected price of equity and the expected price of the privatized asset are sufficiently highly correlated (that is, β is sufficiently close to 1).[3]

[3] This reduction is obscured by the standard system of government financial statistics (GFS) in which the returns from asset sales are treated as revenue (or sometimes negative expenditure) in the year in which sales take place. Thus, from the viewpoint of ministers and officials relying on the GFS statistics, privatization always yields a short-run improvement in the government's financial position

And by substituting (17) into (16) we see that overall ex ante welfare (with respect to the true distribution) is necessarily lower whenever $\beta \geq 0$ since

$$\mathrm{E}\left[\Delta\widehat{W}\right] = -\frac{\gamma\lambda^2\sigma_d^2 x\,(2\beta + x)}{2\,(1+r)^2} - \frac{(1-\lambda)\,(1+r)^2\left((1-\beta)\,(d^*)^2 + (1+\beta)\,\sigma_d^2\right)}{2r\lambda\gamma\,(1+\beta)\,\sigma_d^2}$$

In view of the assumption of constant absolute risk aversion, lump sum transfers between individuals will not affect asset demands and equilibrium prices. Hence, we obtain our main result.

Proposition 1 *Suppose $\beta \geq 0$ and that lump-sum transfers are feasible. Then there exists a set of lump-sum transfers that Pareto-dominate privatization, in terms of individual ex ante welfare, evaluated with respect to the true distribution.*

The converse holds if $\beta < 0$, x is small enough that the correlation of the new asset with market returns remains negative after privatization and λ, the proportion of noise traders, is sufficiently close to 1. In these circumstances, partial privatization, combined with appropriate lump-sum transfers, will yield an ex ante Pareto-improvement.

3 Concluding comments

Despite the existence of voluminous bodies of literature on the equity premium puzzle and on privatization, very little consideration has been given to the link between these two issues. Yet the existence of a large and anomalous risk premium for equity might be expected to have significant implications for a policy program that involves a substantial increase in private holdings of equity, and a corresponding reduction in holdings of debt. The nature of these implications depends, among other things, on the resolution of the equity premium puzzle, that is, on the factors that generate the risk premium for equity.

In this paper, we have considered one popular explanation, the 'noise trader risk' model of De Long et al. In this model, the misperceptions held by noise traders create risk which has real social costs, reflected in the risk premium for equity.[4]

As we have shown, the creation of additional equity through privatization may either exacerbate or mitigate the effects of noise trader risk. The crucial issue is the correlation between noise trader misperceptions of existing private equity and of the equity newly created by privatization. If this correlation is negative, privatization will create opportunities for diversification and may reduce risk. If it is positive, privatization will increase total risk and reduce welfare.

[4] Since each generation of young traders is endowed with labor income, noise traders cannot be driven to extinction through the operation of market forces such as occurs in Blume and Easley (1992). Extinction might take place in a model with infinitely-lived traders. In a setting where noise traders' misperceptions do not affect equilibrium prices, however, De Long et al. (1991) present a model in which long lived noise traders survive or even come to dominate the long-run wealth distribution. Whether such a result is still possible if equilibrium prices are affected by noise traders' misperceptions, is to our knowledge still an open question.

This result must be qualified by the observation that privatization has a range of effects, for example on the operational efficiency of firms, that are not captured in the present model. Similarly it seems likely that misperception of asset returns by noise traders is one of a number of factors that contribute to the emergence of an anomalously large risk premium. A complete evaluation of the welfare effects of privatization, and of the relationship between welfare and the risk premium for equity, requires consideration of all of these factors.

Appendix

To derive the change in the certainty equivalent consumption of a sophisticated consumer who will consume in period t as a result of the privatization, we can utilize (5), the sophisticated trader's asset demand function, and the expressions for the equilibrium prices (i.e. (9) and (10)) to obtain

$$\begin{aligned}\Delta\,\mathrm{CE}_t^I &= \left[r\mathbf{1}+\boldsymbol{\mu}_{t-1}-(1+r)\,\mathbf{p}_{t-1}\right]^{\mathrm{T}}\mathbf{q}_I-\frac{\gamma}{2}\mathbf{q}_I^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{q}_I-\left(r+\mathrm{E}_{t-1}\left[\overline{p}_t^e\right]-(1+r)\,\overline{p}_{t-1}^e\right)\overline{q}_I^e \\ &\quad+\frac{\gamma}{2}\frac{\lambda^2\sigma_d^2}{(1+r)^2}\left(\overline{q}_I^e\right)^2-(1+r)\,\Delta\tau_{t-1}\end{aligned}$$

$$=\frac{1}{2\gamma}\left(\left[r\mathbf{1}+\boldsymbol{\mu}_{t-1}-(1+r)\,\mathbf{p}_{t-1}\right]^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\left[r\mathbf{1}+\boldsymbol{\mu}_{t-1}-(1+r)\,\mathbf{p}_{t-1}\right]-\frac{(1+r)^2}{\lambda^2\sigma_d^2}\left(r+\mathrm{E}_{t-1}\left[\overline{p}_t^e\right]-(1+r)\,\overline{p}_{t-1}^e\right)^2\right)$$
$$-(1+r)\,\Delta\tau_{t-1}.$$

Since

$$\begin{aligned}(1+r)\,\overline{p}_{t-1}^e &= 1+r+\lambda\left(d_{t-1}^e-d^*\right)+\frac{\lambda d^*(1+r)}{r}-\frac{\gamma\lambda^2\sigma_d^2}{r(1+r)} \\ \text{and}\ \ \mathrm{E}_{t-1}\left[\overline{p}_t^e\right] &= 1+\frac{\lambda d^*}{r}-\frac{\gamma\lambda^2\sigma_d^2}{r(1+r)^2},\end{aligned}$$

it follows that,

$$r+\mathrm{E}_{t-1}\left[\overline{p}_t^e\right]-(1+r)\,\overline{p}_{t-1}^e=\frac{\gamma\lambda^2\sigma_d^2}{(1+r)^2}-\lambda d_{t-1}^e,$$

and from (7) we have

$$r\mathbf{1}+\boldsymbol{\mu}_{t-1}-(1+r)\,\mathbf{p}_{t-1}=\gamma\Sigma\begin{pmatrix}1\\x\end{pmatrix}-\lambda\begin{pmatrix}d_{t-1}^e\\d_{t-1}^{ne}\end{pmatrix},$$

Let $k=\lambda^2\sigma_d^2/(1+r)^2$ and notice that

$$\Sigma=k\begin{bmatrix}1&\beta\\\beta&1\end{bmatrix}\ \text{and}\ \boldsymbol{\Sigma}_t^{-1}=\frac{1}{k\left(1-\beta^2\right)}\begin{bmatrix}1&-\beta\\-\beta&1\end{bmatrix},$$

hence we have $2\gamma\left(\Delta\,\mathrm{CE}_t^I+(1+r)\,\Delta\tau_{t-1}\right)$

$$\begin{aligned}
&= \gamma^2\begin{bmatrix}1 & x\end{bmatrix}\Sigma\begin{pmatrix}1\\ x\end{pmatrix}-2\gamma\lambda\begin{bmatrix}1 & x\end{bmatrix}\Sigma\Sigma^{-1}\begin{pmatrix}d_{t-1}^e\\ d_{t-1}^{ne}\end{pmatrix}+\lambda^2\begin{bmatrix}d_{t-1}^e & d_{t-1}^{ne}\end{bmatrix}\Sigma^{-1}\begin{pmatrix}d_{t-1}^e\\ d_{t-1}^{ne}\end{pmatrix}\\
&\quad -\frac{1}{k}\left(\gamma k-\lambda d_{t-1}^e\right)^2\\
&= \gamma^2 k\left(1+2\beta x+x^2\right)-2\gamma\lambda\left(d_{t-1}^e+xd_{t-1}^{ne}\right)+\frac{\lambda^2}{k\left(1-\beta^2\right)}\left(\left(d_{t-1}^e\right)^2-2\beta d_{t-1}^e d_{t-1}^{ne}+\left(d_{t-1}^{ne}\right)^2\right)\\
&\quad -\gamma^2 k+2\gamma\lambda d_{t-1}^e-\frac{\lambda^2}{k}\left(d_{t-1}^e\right)^2\\
&= \gamma^2 kx\left(2\beta+x\right)-2\gamma\lambda x d_{t-1}^{ne}+\frac{\lambda^2}{k\left(1-\beta^2\right)}\left(\beta d_{t-1}^e-d_{t-1}^{ne}\right)^2
\end{aligned}$$

Hence

$$\Delta\,\mathrm{CE}_t^I=\lambda x\left[\frac{\gamma\lambda\sigma_d^2}{(1+r)^2}\left(\beta+\frac{x}{2}\right)-d_{t-1}^{ne}\right]+\frac{(1+r)^2\left(\beta d_{t-1}^e-d_{t-1}^{ne}\right)^2}{2\gamma\sigma_d^2\left(1-\beta^2\right)}-(1+r)\,\Delta\tau_{t-1}.$$

Similarly, for a Noise-trader who is old in period t we have

$$\begin{aligned}
&\Delta\,\mathrm{CE}_t^N-(1+r)\,\Delta\tau_{t-1}\\
&= \left[r\mathbf{1}+\boldsymbol{\mu}_t-(1+r)\,\mathbf{p}_t+\mathbf{d}_t\right]^{\mathrm{T}}\mathbf{q}_N-\frac{\gamma}{2}\mathbf{q}_N^{\mathrm{T}}\boldsymbol{\Sigma}_t\mathbf{q}_N-\left(r+\mathrm{E}_t\left[\overline{p}_{t+1}^e\right]-(1+r)\,\overline{p}_t^e+d_t^e\right)\overline{q}_N^e+\frac{\gamma}{2}\frac{\lambda^2\sigma_d^2}{(1+r)^2}\left(\overline{q}_N^e\right)^2
\end{aligned}$$

Notice that

$$\begin{aligned}
r+\mathrm{E}_{t-1}\left[\overline{p}_t^e\right]-(1+r)\,\overline{p}_{t-1}^e+d_t^e &= \frac{\gamma\lambda^2\sigma_d^2}{(1+r)^2}+(1-\lambda)\,d_{t-1}^e,\\
\text{and } r\mathbf{1}+\boldsymbol{\mu}_{t-1}-(1+r)\,\mathbf{p}_{t-1}+\mathbf{d}_t &= \gamma\Sigma\begin{pmatrix}1\\ x\end{pmatrix}+(1-\lambda)\begin{pmatrix}d_{t-1}^e\\ d_{t-1}^{ne}\end{pmatrix}.
\end{aligned}$$

So $2\gamma\left(\Delta\,\mathrm{CE}_t^N+(1+r)\,\Delta\tau_{t-1}\right)$

$$\begin{aligned}
&= \gamma^2\begin{bmatrix}1 & x\end{bmatrix}\Sigma\begin{pmatrix}1\\ x\end{pmatrix}+2\gamma(1-\lambda)\begin{bmatrix}1 & x\end{bmatrix}\Sigma\Sigma^{-1}\begin{pmatrix}d_{t-1}^e\\ d_{t-1}^{ne}\end{pmatrix}+(1-\lambda)^2\begin{bmatrix}d_{t-1}^e & d_{t-1}^{ne}\end{bmatrix}\Sigma^{-1}\begin{pmatrix}d_{t-1}^e\\ d_{t-1}^{ne}\end{pmatrix}\\
&\quad -\frac{1}{k}\left(\gamma k+(1-\lambda)\,d_{t-1}^e\right)^2\\
&= \gamma^2 k\left(1+2\beta x+x^2\right)+2\gamma(1-\lambda)\left(d_{t-1}^e+xd_{t-1}^{ne}\right)+\frac{(1-\lambda)^2}{k\left(1-\beta^2\right)}\left(\left(d_{t-1}^e\right)^2-2\beta d_{t-1}^e d_{t-1}^{ne}+\left(d_{t-1}^{ne}\right)^2\right)\\
&\quad -\gamma^2 k-2\gamma(1-\lambda)\,\lambda d_{t-1}^e-\frac{(1-\lambda)^2}{k}\left(d_{t-1}^e\right)^2\\
&= \gamma^2 kx\left(2\beta+x\right)+2\gamma(1-\lambda)\,\lambda x d_{t-1}^{ne}+\frac{(1-\lambda)^2}{k\left(1-\beta^2\right)}\left(\beta d_{t-1}^e-d_{t-1}^{ne}\right)^2
\end{aligned}$$

Hence

$$\Delta\,\mathrm{CE}_t^N = x\left[\frac{\gamma\lambda^2\sigma_d^2}{(1+r)^2}\left(\beta+\frac{x}{2}\right)+(1-\lambda)\,d_{t-1}^{ne}\right]+\frac{(1-\lambda)^2\,(1+r)^2\left(\beta d_{t-1}^{e}-d_{t-1}^{ne}\right)^2}{2\lambda^2\gamma\sigma_d^2\left(1-\beta^2\right)}-(1+r)\,\Delta\tau_{t-1}.$$

References

Blume Lawrence, E., and Easley, David. "Evolution and Market Behavior." *J. Econ. Theory* 58(1) (1992): 9-40.

De Long, J. Bradford, Shleifer, Andrei, Summers, Lawrence, and Waldmann, Robert J. "Noise Trader Risk in Financial Markets." *J.P.E.* 98 (Aug. 1990): 703-738.

De Long, J. Bradford, Shleifer, Andrei, Summers, Lawrence, and Waldmann, Robert J. "The Survival of Noise Traders in Financial Markets." *J. Business* 64(1) (1991): 1-19.

Hirshleifer, Jack, and Riley, John. *The Analytics of Uncertainty and Information*, Cambridge, UK: Cambridge University Press, 1992.

Kikeri, S.; Nellis, J.; Shirley, M., *Privatization: The Lessons of Experience*, Washington, DC, The World Bank (1992).

Kocherlakota, Narayana R., "The Equity Premium: It's Still a Puzzle," *J. Econ. Lit.* 34 (March 1996): 42–71.

Mehra, Ranjesh and Prescott, Edward C. "The Equity Premium: a Puzzle", J. Monetary Econ. 15 (March 1985): 145–61.

Orosel, G.O., "Informational Efficiency and Welfare in the Stock Market." *European Econ. Rev.* 40(7) (Aug. 1996): 1379–1411.

Palley, T.,"Speculation and Tobin taxes: Why Sand in the Wheels can Increase Economic Efficiency." *Zeitschrift Fur Nationalokonomie,* 69 (2) (1999): 113–126.

Quiggin, John, "Does Privatization Pay?", *Aust. Econ. Rev* 95 (June 1995), 23–42.

Shiller, Robert J., *Market Volatility*, Cambridge, MA: The MIT Press, 1989.

Vickers, John, and Yarrow, George. *Privatization: An economic analysis*, Cambridge, Mass.: MIT Press, 1988.